

Implementation of a current transformer model in Simulink® based on the Hysteresis Theory of Gilet-Eserton

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1 INTRODUCTION

Correct and predictable performance of current transformers (CTs) is crucial for reliability of protective relays connected to these CTs. The straightforward testing of HV and EHV current transformers together with protective relays is constrained by the size and capabilities of test devices, which does not allow using the primary current injection methods in most practical situations. Therefore, manufacturers of numerical protection IEDs have been using various CT models for a long time.

The accuracy of these models has an immediate impact on security and dependability of protection devices. For example, if a specific CT model does not reproduce some important physical phenomena, then a saturation detector or a waveform recovery algorithm developed and tested using this model might fail in a real application.

CT models based on Jiles-Atherton theory of hysteresis have become an industry standard since they were implemented in RTDS® and PSCAD® systems. These models are essentially fixed “CTs + leads + burden” constructs, because the secondary current waveform is obtained as a solution to a set of combined equations describing both the internal CT physics and the electrical network of the burden [1]. Such an approach naturally limits the range of available CT schemes and burden options.

For example, one might need to model the following protection scheme (admittedly, uncommon):

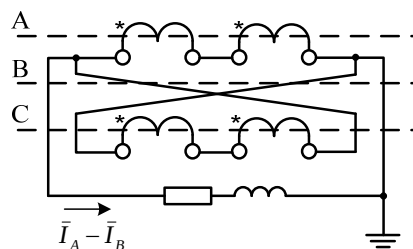


Figure 1: Complex CT arrangement

In this scheme the serial connection of the CTs increases their combined effective knee-point voltage, while the subtractive connection of phases A and C allows protecting an MV feeder from phase-to-phase faults using a relay with a single analogue input (the sensitivity, obviously, depends on the faulty phases).

Another more common example is a high-impedance busbar protection, where the burden includes a non-linear resistor. For such a scheme a new “CTs + burden” module would have to be added in RTDS® or PSCAD®, especially considering the fact that non-linear resistors from different manufacturers can have different characteristics.

In this paper a Simulink® model of CT is presented, which is based on Jiles-Atherton theory of hysteresis from [2] with minor adjustments. The main advantage of this model is that it is decoupled from the burden network. Hence, using Simulink®, several CT models can be interconnected together with other electrical elements from SymPowerSystems library in any imaginable combination.

2 JILES-ATHERTON (J-A) MODEL OF FERROMAGNETIC HYSTERESIS

2.1 Jiles-Atherton (J-A) model of ferromagnetic hysteresis

The model was originally proposed by D. C. Jiles and D. L. Atherton in their paper [3]. Unlike the phenomenological Preisach model [4], it is based on the physics of the ferromagnetic materials, namely on energy dissipation within a magnetic material resulting from changes in its magnetization. A comparison between Preisach and J-A models of hysteresis can be found in [5]. The authors demonstrated the similar performance of both approaches; however the J-A hysteresis model offers a higher computational efficiency according to [6].

An interpretation of the J-A model adopted in this paper generally follows the existing implementation in RTDS® and PSCAD®, which in turn is based on [2]. The following enhancements to the original J-A model formulation in [3] were proposed in [2]:

- an improved anhysteretic function instead of the Langevin function;
- dynamic adjustment of the Domain Pinning Parameter.

Equations (1-10) briefly summarize the variation of the J-A model presented in [2].

$$B = \mu_0(M + H), \quad (1)$$

where B – magnetic flux density; M – magnetization; H – magnetic field intensity.

$$\text{If } (M_{an} - M)\delta \geq 0: \quad (2)$$

$$\frac{dM}{dH} = \frac{c \frac{dM_{an}}{dH_e} + \frac{M_{an} - M}{\delta k_{mod} - \frac{\alpha(M_{an} - M)}{1 - c}}}{1 - \alpha c \frac{dM_{an}}{dH_e}}, \quad (3)$$

otherwise:

$$\frac{dM}{dH} = \frac{c \frac{dM_{an}}{dH_e}}{1 - \alpha c \frac{dM_{an}}{dH_e}}, \quad (4)$$

where M_{an} – anhysteretic magnetization; α , c – J-A model parameters.

$$\delta = \text{sign}\left(\frac{dH}{dt}\right), \quad (5)$$

$$M_{an} = M_s \frac{a_1 H_e + H_e^b}{a_3 + a_2 H_e + H_e^b}, \quad (6)$$

$$H_e = H + \alpha M, \quad (7)$$

$$\text{If } H\delta \geq 0: \quad (8)$$

$$k_{mod} = k \left(1 - \beta \left(\frac{M}{M_s} \right)^2 \right), \quad (9)$$

otherwise

$$k_{mod} = k. \quad (10)$$

where k – J-A model parameter, β – coefficient of k adjustment.

Now let us adapt the equations (1-10) for efficient implementation in the Simulink® environment.

First of all, condition (8) is replaced by (11), the explanation given in section 3.1 of this paper.

$$\text{If } M\delta \geq 0 : \quad (11)$$

The original equation (6) for anhysteretic magnetization M_{an} is only valid for positive values of H_e and M_{an} , therefore it needs to be modified as follows:

$$M_{an} = \text{sign}(H_e) \cdot M_s \frac{a_1|H_e| + |H_e|^b}{a_3 + a_2|H_e| + |H_e|^b}. \quad (12)$$

The derivative of M_{an} with respect to H_e (used in (3) and (4)) can be also calculated explicitly as per [2]:

$$\frac{dM_{an}}{dH_e} = M_s \frac{a_1a_3 + ba_3|H_e|^{b-1} + (b-1)(a_2 - a_1)|H_e|^b}{(a_3 + a_2|H_e| + |H_e|^b)^2}. \quad (13)$$

Equations (3, 4) represent continuous-time relationship between magnetization M and magnetic field intensity H . Implementing this model in Simulink® requires its discrete-time modification (the time step can be either variable or fixed).

At any step in time the input variables for the model are:

- the present value of the instantaneous excitation voltage v_e ;
- the present value of the magnetization M ;
- the present value of the magnetic field intensity H and its previous value H_{PREV} .

Using the discrete values of H and H_{PREV} equation (5) can be replaced by (14):

$$\delta = \text{sign}(H - H_{PREV}). \quad (14)$$

The model's output variables are:

- time derivative of the magnetic field intensity dH/dt ;
- time derivative of the magnetization dM/dt .

Several ancillary equations are used in the process:

$$dB/dt = \frac{v_e}{N_2 \text{ Area}}, \quad (15)$$

$$dH/dt = \frac{dB/dt}{\mu_0 \left(\frac{dM}{dH} + 1 \right)}, \quad (16)$$

$$dM/dt = \frac{dB/dt}{\mu_0} - dH/dt. \quad (17)$$

The obtained time derivatives of the magnetic field intensity and magnetization are subsequently passed through integrators, thus determining the transition to the next time step.

Having calculated the value of the magnetic field intensity H , the secondary current i_2 to be injected into the CT burden network can be found using the following equation:

$$i_2 = i_1 \frac{N_1}{N_2} - H \frac{\text{Length}}{N_2}. \quad (18)$$

A flowchart in Fig. 2 shows the sequence of calculations executed at every step in time.

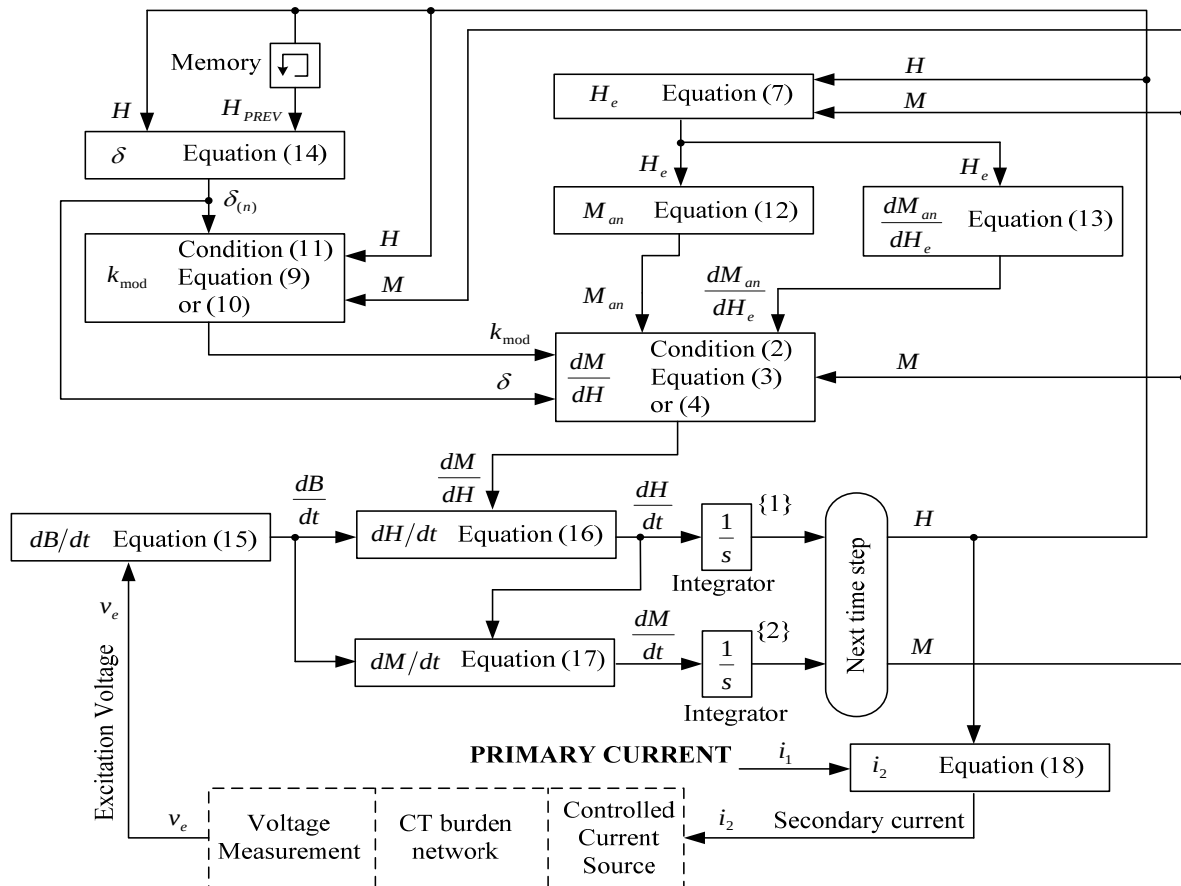


Figure 2: CT model flowchart

The two integrators {1} and {2} in the flowchart are critically important for the Simulink® implementation of J-A hysteresis model. It would be possible to implement the same equations in Simulink® without integrators but this would most likely lead to the occurrence of multiple algebraic loops (for a more detailed description of algebraic loops in Simulink® see [7]). The in-built Simulink® loop solver is generally robust, but sometimes it may not converge and also significantly reduces the simulation speed.

The integrators effectively break the direct feedthrough links and therefore eliminate the algebraic loops. The calculations at each time step are thus reduced to a number of sequentially executed instructions, speeding up the simulation and ensuring the model's convergence.

3 PROBLEMS FACED AND SOLUTIONS PROPOSED

3.1 Adjustment of the Domain Pinning Parameter “k”

The original condition (8) for k adjustment was first taken from [2] and implemented in Simulink® “as is”. This led to several cases of poor numerical performance of the model, namely inaccurate results (with high **Relative Tolerance** setting of Simulink® solver, 1e-4) or unacceptably slow simulation speed (with the same tolerance parameter set to lower value, 1e-7).

After some investigations the cause of such poor performance was pinned down to the sudden change of the derivative dM/dH (see the breaking point in Fig. 3), which is evidently non-physical. The proposed condition (11), on the contrary, does not result in such undesirable behaviour of M - H function, while still doing the job for which the adjustment of k was introduced in the first place – it reduces the width of the simulated hysteresis loop in the shoulder area [2].

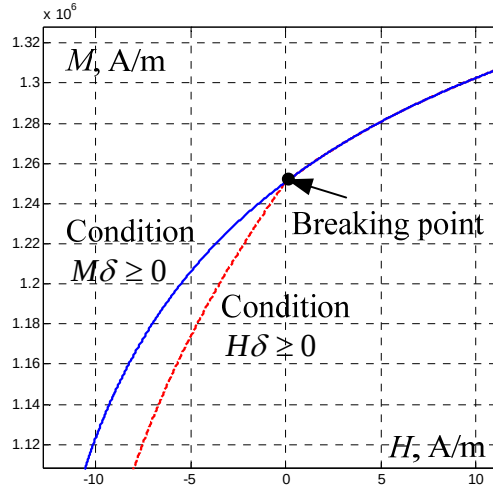


Figure 3: M-H characteristics with different adjustment conditions

It is worth mentioning that in [2] the authors proved that the dynamic adjustment of k “does not make any noticeable effect on the secondary current waveform”. However, this adjustment was preserved in the CT model described here (with condition (8) replaced by (11)) in order to unify it with existing models in RTDS® and PSCAD®.

3.2 The model's behaviour under heavy saturation conditions.

It can be seen from the flowchart in Fig. 2 that the values of H and M are calculated by integrating their respective time derivatives. When a CT model is pushed into heavy saturation, the integrators become prone to numerical errors, see Fig. 4 and Fig. 5. This can have two effects on the CT simulation, both adverse:

- the errors may accumulate, yielding the simulation results completely invalid;
- the irregular shape of the M - H curve slows the calculations down in the **Continuous** simulation mode (see the first line in Table 2).

The first problem can be overcome by reducing the **Relative Tolerance** setting to $1e-7$, but this, unfortunately, makes the simulation speed much slower (see the second line in Table 3).

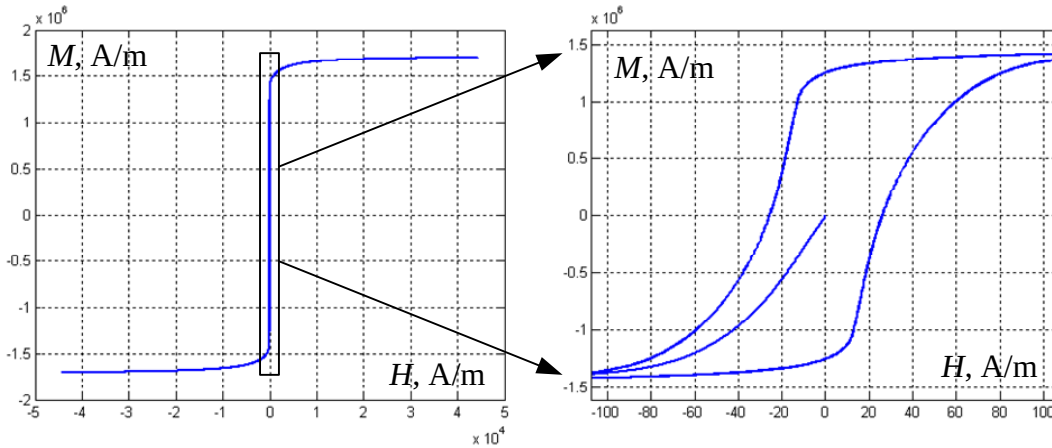


Figure 4: M-H hysteresis loop under heavy saturation

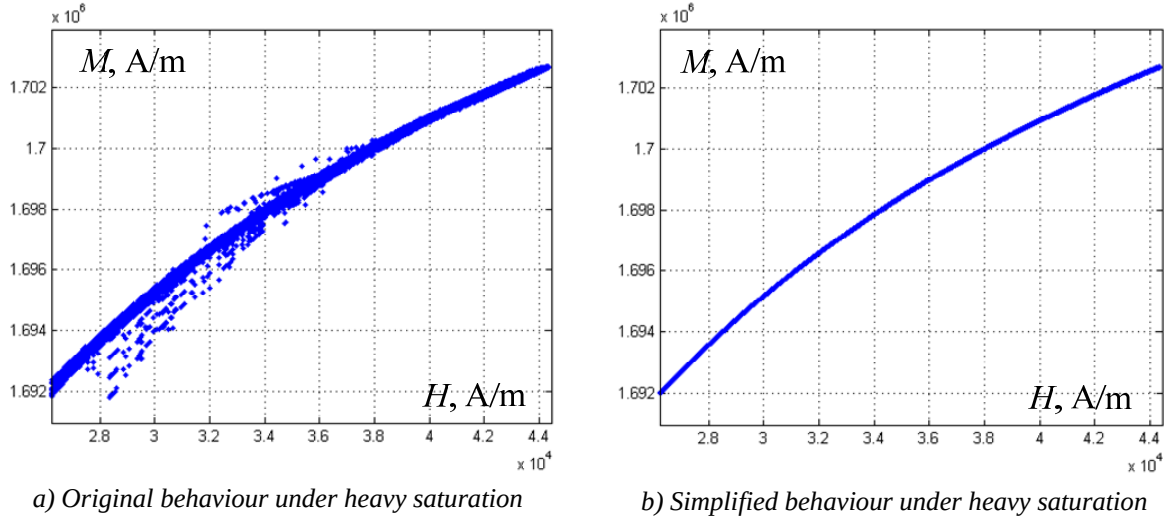


Figure 5: M-H characteristics under heavy saturation

At the same time it is obvious that once the saturation reaches a certain limit, H_s , the difference between ascending and descending branches of the hysteresis loop becomes negligible (see Fig. 4). Hence switching to a simplified anhysteretic model can be introduced as described in equations (19-21).

$$\text{If } |H| \geq H_s : \quad (19)$$

$$M = M_{an} , \quad (20)$$

$$\frac{dM}{dH} = \frac{dM_{an}}{dH} = \frac{\frac{dM_{an}}{dH_e}}{1 - \alpha \frac{dM_{an}}{dH_e}} . \quad (21)$$

Equation (20) resets the corresponding integrator {2} in the flowchart Fig. 2 and, therefore, eliminates the numerical error seen in Fig. 5.a.

The fact that M_{an} is a function of H_e (12), while H_e itself is a function of M (7), makes obtaining a close-form solution of M with respect to H a challenging task.

A simple iterative procedure was proposed to calculate M based on a known value of H :

1. Assume $M = M_s$,
2. Calculate H_e using equation (7),
3. Calculate new value of $M = M_{an}$ using equation (12),
4. Repeat steps 2-3 N times.

Table 1 below shows that after maximum two iterations the error becomes negligible.

Iteration number	H = 1000 A/m $M_{true} = 1518043.45$ A/m		H = 2000 A/m $M_{true} = 1559462.85$ A/m		H = 20000 A/m $M_{true} = 1684462.19$ A/m	
	M_{iter}	Error, %	M_{iter}	Error, %	M_{iter}	Error, %
0	1720000	13.3	1720000	10.3	1720000	2.1
1	1518188.59	9.6e-3	1559530.88	4.4e-3	1684462.90	4.3e-5
2	1518043.55	6.9e-6	1559462.88	1.8e-6	1684462.19	0
3	1518043.45	0	1559462.85	0	1684462.19	0

Table 1: Accuracy of iterations

The simplified saturation behaviour is implemented as a configurable option that significantly increases the simulation speed. The default threshold H_s is 2000 A/m (based on the default hysteresis parameters from [2]), the corresponding relative difference between the major loop branches is $(1.5606e+6 - 1.5592e+6) / 1.5592e+6 * 100 = 0.09\%$

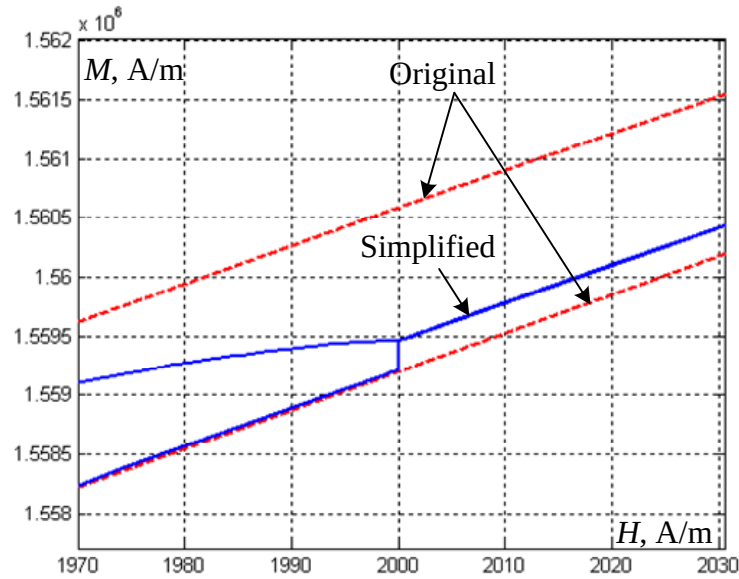


Figure 6: M-H characteristics under heavy saturation

A comparison between different simulation modes can be found in the table below. Timing tests were performed using a laptop with Intel® Core™2 Duo CPU P8400 @ 2.26GHz with 2GB of RAM.

Simplified saturation behaviour	Relative Tolerance	Accuracy	Execution time (per one second of simulation)
Disabled	1e-4	Bad	1.16 s
Disabled	1e-7	Good	1.96 s
Enabled	1e-4	Good	0.59 s

Table 2: Simulation results

3.3 Connecting the CT model to inductive burden.

The CT model in Simulink® uses a controlled current source as its output, which is consistent with the MATLAB Help guidelines for modelling of non-linear elements [8]. A current source, obviously, cannot be connected in series with an inductor. To allow connecting non-linear elements to inductors MATLAB Help recommends paralleling the current source with a relatively high resistance; that is why the **R_Loss** resistor is introduced in the scheme below.

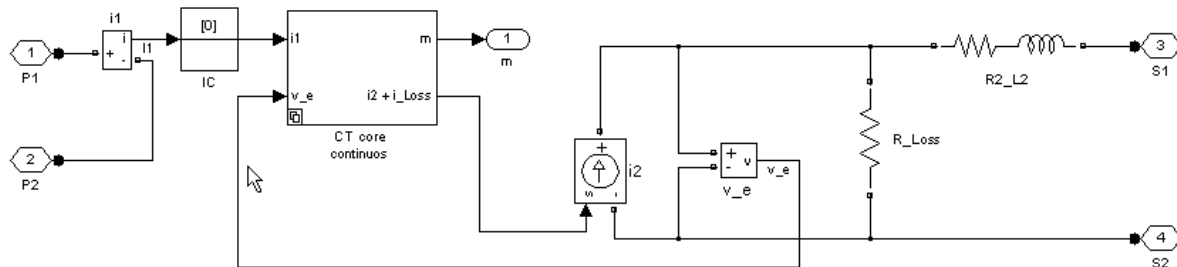


Figure 7: Internal connections of the CT model

The recommended value of the **R_Loss** resistance is about 1000-10000 times the total burden impedance. This would introduce an additional error of 0.01-0.1% that can be ignored in real-life scenarios.

4 UTILIZING SIMULINK® CT MODELS – PRACTICAL EXAMPLES

4.1 Implementation of the scheme from Fig. 1.

The example below shows the implementation of the scheme described in section 1 “INTRODUCTION” and proves the idea of decoupling CT model from its burden. The *RL* burden in Fig. 8 can be easily replaced with any kind of custom-built nonlinear electrical elements.

The primary and secondary current waveforms in Fig. 9 (measured at one of the CTs) clearly demonstrate the phenomenon of saturation.

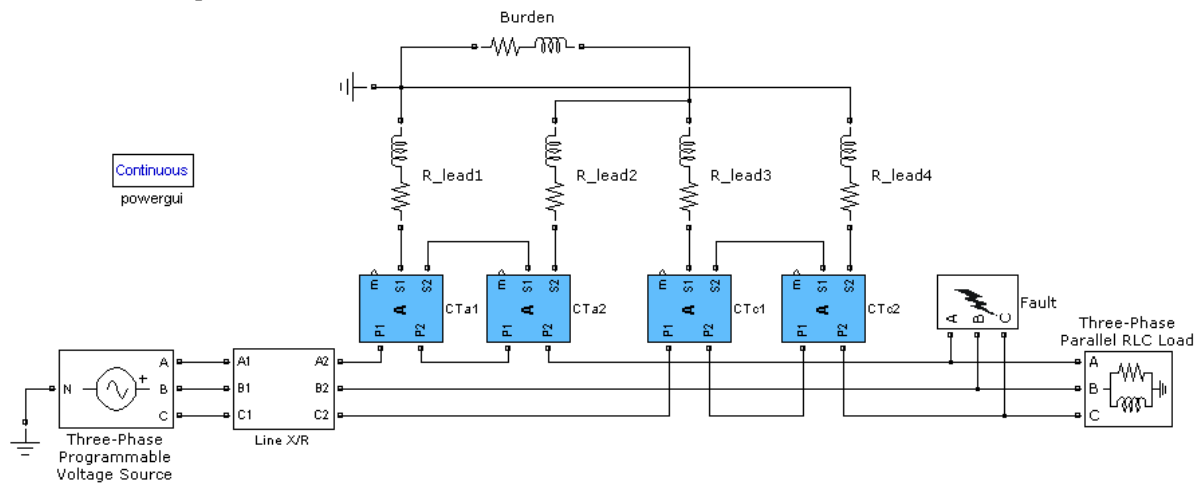


Figure 8: Implementation of a complex CT scheme

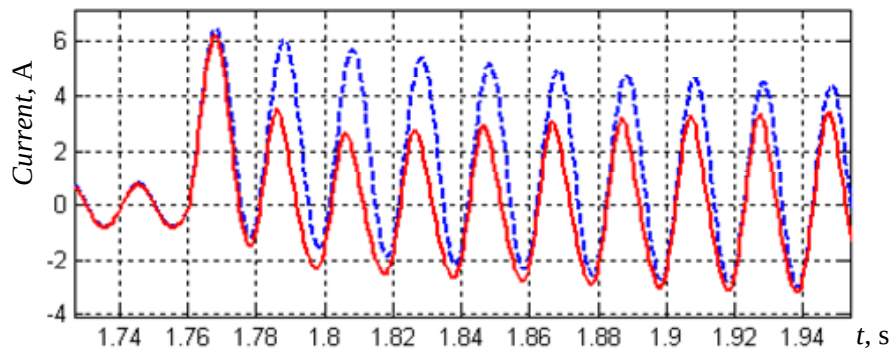


Figure 9: Illustration of saturation in the complex CT scheme

4.2 Using CT models in the development of new protection algorithms

According to [9] and [10], Simulink® is now becoming the main tool for the development of new protection and control algorithms. Though the final stage of such development process is still Hardware-In-the-Loop (HIL) testing, the ability to test those algorithms within Simulink® environment is absolutely critical for fast delivery of the project.

Testing protection relays for CT requirements is extremely time-consuming. A brief description of the testing procedure adopted in ALSTOM GRID and of the efforts involved can be found in [11]. In that particular project of Current Differential protection with Transient Biasing the Simulink® CT model was used extensively. This helped to significantly speed up the development by refining the algorithm even before it was actually implemented in the protection device.

Obviously, testing new protection algorithms with CT models in Simulink® does not take into account many hardware-related factors, and as such needs to be considered as preparatory to RTDS®

testing. For example, during the development described in [11] thousands of faults with various characteristics were first injected into Simulink® CT models connected to the model of the relay. Only when the algorithm's performance was considered satisfactory, the project proceeded into the final HIL testing using RTDS® equipment.

5 CONCLUSION

The CT model presented in this paper was created to facilitate the development (and especially validation) of new protection algorithms in Simulink®. The main goals were to make this new model as flexible, accurate and fast as possible without sacrificing its unification with existing CT models in RTDS® and PSCAD®. A well-proven variation of J-A theory of ferromagnetic hysteresis was taken from [2] and subjected to some minor modifications. The resulting Simulink® CT model was successfully used in two recent development projects – the Transient Biasing for Current Differential mentioned above (already available in ALSTOM GRID product range) and another Current Differential improvement that is now going through its final testing stages.

The major constraint of the developed CT model is that it needs to be executed in **Continuous** simulation mode (**powergui** block setting) and, therefore, the solver's type has to be **Variable-step**. The simulation speed in **Continuous** mode is sufficient for small protection scheme containing 1-10 CTs. For example, the scheme with 4 CTs presented in Fig. 8 takes 0.66 seconds of real time per 1 second of simulation (check section 3.2 for details on the PC used). However, for large schemes with tens of CTs connected to a common burden network the simulation speed is not satisfactory.

One way of improving the model might be switching to **Discrete** simulation mode and **Fixed-step** solver type. So far the author's attempts to do so have not succeeded – the discrete variation of the same Simulink® model exhibited poor accuracy and low reliability, especially with inductive burden connected.

Another direction for future work is implementation of the J-A hysteresis model in a universal electromagnetic element using Simulink® SimScape™. This universal element would represent the relations between its winding's electrical parameters and its core's magnetic parameters. Several such elements could be connected together (meaning electrical connection of windings and magnetic connection of cores/gaps) to model any complex multi-winding/multi-core electromagnetic system – transformer, autotransformer, reactor etc.

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